QUANTUM MECHANICS
IN YOUR FACE

Sidney Coleman

Harvard University

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Transcribed by Andrew Kobach

Forward

For many years I have been advising students, graduate and undergraduate, to watch a video of Sidney Coleman's classic presentation, *Quantum Mechanics In Your Face*. I will continue to do that, because everyone should see the master at work. But Andrew Kobach has done a great service by providing a transcript. Listening to Sidney, it is easy to get so mesmerized by the perfection of his lecture that you miss the precision of his logic. Following along in the transcript helps!

Prof. Howard Georgi
Cambridge, MA

Preface

The problem with quantum mechanics is not wave-particle duality, not superposition, not non-commuting observables, not entanglement, and not even the measurement problem. In fact, there is no problem. The difficulty we have with quantum mechanics is that the universe uses a different theory of probability than the one we use. This fact has caused excessive anxiety for us humans, and many have worried about the "interpretation of quantum mechanics," attempting to grab onto anything familiar. Sidney Coleman had an unmistakable message: *the problem is not quantum mechanics, the problem is us, stupid!*

This is supposed to be an honest, but not perfect, transcription of the video recording of Coleman's 1994 lecture at the Spring Meeting of the New England Section of the American Physical Society. I had to make some choices regarding punctuation, footnotes, some section titles, how to fill in the gaps when the audio cuts out, and I reduced the number of sentences that begin with "Now." The contents of the transparencies were interwoven into the text for a more seamless reading experience. (An alternative could have been to try in vain to reproduce them as figures and have the text punctuated with awkward phrases like, "Please see Fig. 2, which just has more text.")

I am grateful to Aneesh Manohar for introducing me to *Quantum Mechanics In Your Face*, Howard Georgi, Ken Intriligator, Shauna Kravec, and John McGreevy for providing invaluable support and feedback, and Diana Coleman for granting permission to post this document as a 3rd party on arXiv.org.

Coleman's lecture can be difficult to appreciate when watching the video – a similar difficulty, I imagine, for someone roller skating through the Louvre. I hope reading Coleman's words can make it easier to appreciate this priceless gem.

Andrew Kobach, Ph.D.
San Diego, CA

Introductory Remarks

This lecture has a history. It's essentially a rerun of a lecture I gave as the Dirac Lecture at Cambridge University a little under a year ago. There's a story there. I had been asked to give this lecture several years ago. It was two years in the future and, of course, when someone asks you to do something two years in the future, that's never – you always say yes. And the time came I got a communication from Peter Goddard at St John's College, who was running the operation, who said, "What do you want to talk about?" I said, "I don't know. Who's the audience?" And he said, "Oh, it's pretty mixed. You get physics graduate students, physics undergraduates, people from chemistry and philosophy and mathematics." I thought...*mm-mm*...these are not the people to address on the subject of non-abelian quantum hair on black holes (which is what I was working on at the moment). So I said, "Look, I've always been interested in giving a lecture on quantum mechanics, what a strange thing it is, and exactly what strange thing it is, and do you think such a lecture would be suitable?" And he said, "Yes. Give us a clever title." So I emailed back

Quantum Mechanics In Your Face,

because I wanted to really confront people with quantum mechanics. And Peter said, "No good. A British audience would not understand the locution and indeed might think it was obscene." "All to the good!" I said. But, he was adamant. So, I said one of the themes of the proposed lecture was that people get a lot of confusion because they keep trying to think of quantum mechanics as classical mechanics. I suggested this alternative title:

It's Quantum Mechanics, Stupid!

And he said, “Nope! A British audience wouldn’t get it. Too American.” So I said, “Well, alright, if you want something British,”

And Now For Something Completely Different:
Quantum Reality.

He said, “Too facetious.” So finally we settled on this title:

Quantum Mechanics With The Gloves Off,

which you can see is a little wimpier than the others. But now I’m back in the land of free speech, so the title of the talk is *Quantum Mechanics In Your Face*.

The talk will fall into three parts. There will be a preliminary where I give a quick review of quantum mechanics. I would say it was the Copenhagen interpretation, or the interpretation in somebody’s textbook, but it’s not really that, it’s looser and more sloppy. Architectural historians when they’re discussing the kinds of buildings that were being built in a certain place in a certain time, but they aren’t in any particularly well-defined style (it’s just what builders threw up in the United States circa 1948), they call it vernacular architecture. And this will be a quick review of vernacular quantum mechanics. It’s more to establish notation, to make sure we’re all on the same wavelength.

Then the two main parts of the lecture will be, firstly, a review of a pedagogical improvement on John Bell’s famous analysis of hidden variables in quantum mechanics, which is, in fact, easier to explain that Bell’s original argument and deserves to be widely publicized. It was built by David Mermin, out of some earlier work by Greenberger, Horne, and Zeilinger (the GHZM analysis).

Then the second part of the lecture I will turn to the much-vexed question sometimes called the “interpretation of quantum mechanics." Although, as I will argue, that's really a bad name for it.

I want to stress that I have made no original contributions to this subject. There's nothing I will say in this lecture, with the exception of the carefully prepared spontaneous jokes,[1] that cannot be found in the literature. Of course, such is the nature of this subject – there's nothing I will say where the contradiction cannot also be found in the literature. So, I claim a measure a responsibility, if no credit (the reverse of the usual scholarly procedure). Also, I will stick strictly to quantum mechanics in flat space, and not worry about either classical or quantum gravity. We'll have problems enough keeping things straight there, without worrying about what happens when the geometry of spacetime is itself a quantum variable.

A Quick Review of (Vernacular) Quantum Mechanics

To begin with, a very quick review.

(1) The state of a physical system at a fixed time t is a vector in a Hilbert space, following Dirac, we call it $|\psi(t)\rangle$, normalized to unit norm $\langle\psi(t)|\psi(t)\rangle = 1$.

(2) It evolves in time according to the Schrödinger equation:

$$i\frac{\partial}{\partial t}\,|\psi(t)\rangle = H\,|\psi(t)\rangle\,, \tag{1}$$

where the Hamiltonian H is some self-adjoint linear operator – a simple one if we are talking about a single atom,

[1] That was one of them.

a complicated one if we are talking about a quantum field theory.[2]

(3) Some, maybe all, self-adjoint operators are "observables." If the state $|\psi(t)\rangle$ is an eigenstate of the observable A with an eigenvalue a,

$$A\,|\psi(t)\rangle = a\,|\psi(t)\rangle\,, \tag{2}$$

then we say the value of A is certain to be a. Strictly speaking, this is just a definition of what I mean by "observable" and "observed." But of course, that's because those words have not occurred yet in the lecture, so I can call them what I want. But of course, that's like saying Newton's Second Law, $\mathbf{F} = m\mathbf{a}$, as it appears in textbooks on mechanics, is just a definition of what you mean by "force." Of course, that's true, strictly speaking. But we live but in a landscape – there is an implicit promise that when someone writes that down when they begin talking about particular dynamical systems, they will give laws for force, not, say, for some quantity involving the 17th time derivative of the position. Likewise, the words "observable" and "observed" have a history before quantum mechanics. People like to say that *classically*, all these things have a meaning in *classical* mechanics, and we need a *classical* level. But really, it goes way earlier than classical mechanics.[3] There's an implicit promise in here that when you put the whole theory together and start calculating things that these words

[2] If there's anyone who does not understand the material presented so far, please leave the auditorium, because you won't be able to understand anything else in the lecture.

[3] I'm sure the pre-Colombian inhabitants of Massachusetts were capable of saying in their language, "I observe a deer," despite their scanty knowledge of Newtonian mechanics, and indeed I even suspect that the deer was capable of observing the Native Americans, despite its even weaker grasp on action and angle variables.

"observe" and "observable" will correspond to entities that act in the same way as those entities do in the language of everyday speech, under the circumstances of which the language of every speech is applicable. To show that is a long story and not something I'm going to focus on here, involving things like the WKB approximation and von Neumann's analysis of an ideal measuring device, but I just wanted to point out that that's there.

(4) What happens when the state $|\psi(t)\rangle$ is not an eigenstate of observable A? Every measurement of A yields one of the eigenvalues of A. The probability of finding a particular eigenvalue a is

$$\big|P(A;a)\,|\psi(t)\rangle\big|^2, \tag{3}$$

where $P(A;a)$ is the projection operator on the subspace of states with eigenvalue a. I'm assuming here, just for notational simplicity, that the eigenvalue spectrum is discrete. If a has been measured, then the state of the system after the measurement is

$$\frac{P(A;a)\,|\psi(t)\rangle}{\big|P(A;a)\,|\psi(t)\rangle\big|}. \tag{4}$$

This is just that part of the wave function – all the rest of has been annihilated. And of course, it has to be rescaled (or, being a quantum field theorist, I should say, I suppose, *renormalized*), so it has unit norm again. This is the famous projection postulate, sometimes called the reduction of the wave packet. It is very different from the previous three statements I've discussed, because it contradicts one of them: causal time evolution according to Schrödinger's equation. Schrödinger's equation is totally causal: given the initial wave function (the initial state of the system), the final state is completely determined. Furthermore, this causality is time-reversal invariant: given the final state, the initial state is completely determined. This operation [in Eq. (3) and (4)] is something other than Schrödinger's equation. It is not deterministic. It is probabilistic. Not only can you not predict the future from the past, even when you know the future, you don't know what the past was. If I measure an electron and discover it is in an eigenstate of σ_z, with $\sigma_z = 1$, I have no way of knowing what its initial state was. Maybe it was $\sigma_z = 1$, maybe it was $\sigma_x = 1$, and it turned out I was in the branch that got the 50% probability of measuring σ_z up.

I will, in Part Two, return to a critical analysis of the reduction of the wave packet, but for the first of this lecture, I'd like to take it as given.

Better than Bell: the GHZM Effect

There are references for this part, but actually I call them credits, because I notice that nobody ever writes down the references.[4] This whole analysis, as everyone knows, starts with the work of Einstein, Podolsky, and Rosen:

A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47** (1935) 777,

which sat around as an irritant for some years until John Bell, picking up an idea from David Bohm, was able to turn it into a conclusive argument against hidden variables:

J.S. Bell, Rev. Mod. Phys. **38** (1966) 447.

A pedagogical improvement was made by David Mermin:

N.D. Mermin, Physics Today, April 1985, p. 38,

[4] It's just here to avoid the speaker being sued.

who, to my mind at least, really clarified what was going on in Bell's analysis. And then a completely different experiment was suggested by Greenberger, Horne, and Zeilinger. I've got a reference to a paper they wrote with Abner Shimony:

> D. Greenberger, M. Horne, A. Shimony, A. Zeilinger, Am. J. Phys. **58** (1990), 1131,

not because that was the original paper, but the original paper is a brief report in a conference proceedings. [This paper by Mermin discusses a simplified gedanken experiment based on the work of Greenberger, Horne, and Zeilinger:]

> N.D. Mermin, Am. J. Phys. **58** (1990), 731;
> Physics Today, June 1990, p. 9.

[I'll discuss today this gedanken experiment], polishing it up, and it's my version of Mermin's version of Greenberger, Horne, and Zeilinger's gedanken experiment, inspired by John Bell's, based on Bohm and Einstein, Rosen, and Podolsky. And I've left out about 90% of the references that you're going to see now.

The way I like to think about this analysis is by imagining a physicist, who I call Dr. Diehard, who was around at the time of the discovery of quantum mechanics in the late 1920's and didn't believe it. Although some time as passed since then, he's still around, quite old, but intellectually vigorous, and he still doesn't believe in it. Our task is to convince him that quantum mechanics is right, and classical ideas are wrong (or I'll say, primitive, pre-classical ideas). There's no point in trying to wow him with the anomalous magnetic moment of the electron, or the behavior of artificial atoms, or anything like that, because he is so deep down opposed to quantum mechanics, and so old and stubborn, that as soon as you start putting a particular quantum-mechanical equation on the board, his brain turns off.[5] The only way to convince him is on very general grounds, not by doing particular calculations.

For a first thought, you say, easy, quantum mechanics is probabilistic, classical mechanics is deterministic. If I had an electron in an eigenstate of σ_x, and I choose to measure σ_z, I can't tell whether I'm going to get $+1$ or -1. There's no way anyone can tell. That's very different from classical mechanics, and it seems to describe the real world. Well, Dr. Diehard is not convinced for a second by that. He says, "Probability has nothing to do with this fancy quantum mechanics. Jerome Cardan was writing down the rules of probability when he analyzed games of chance in the late Renaissance. When I flip a coin or go to Las Vegas and have a spin on the roulette wheel, the results seem to be perfectly probabilistic, and I don't see any Planck's constant playing any significant role there. It's just like that. The reason the roulette wheel gives me a probabilistic result is that there are all sorts of sensitive initial conditions, which I can't measure well enough, initial conditions to which the final state of the ball is sensitive. There are all sorts of degrees of freedom of the system which I can't control, and because of my ignorance, not because of any fundamental physics, I get a probabilistic result." (This is sometimes called the hidden-variable position.) "Really, you don't know everything about the state of the electron when you measure its momentum and its spin along the x-axis. There are zillions of unknown hidden variables, which you cannot control, and maybe also in the system that is measuring the electron. There is no separation in this viewpoint between the observing system and the quantity being observed. If you knew those quanti-

[5] Rather like my brain in a seminar on string theory.

ties exactly, then you know exactly what the electron was going to do in any future experiment, but since you only know them probabilistically, then you only have a probabilistic distribution. Put in more fancy mathematical notation, consider an observable A is a function of very many 'hidden' variables s:

$$A = A(s), \tag{5}$$

which may involve the 'apparatus' as well as the 'system.' The probability to measure $A = a$ is then

$$\text{Prob}[A = a] = \int \delta(a - A(s))\; d\mu(s), \tag{6}$$

where $d\mu(s)$ is the probability distribution for the hidden variables. [The probabilistic aspect of quantum mechanics is] a result from our ignorance, not some quantum nonsense!"

Well, in fact this is right. You can get probability through classical mechanics. And John von Neumann way back was aware of this and said, "No! That's not the real difference between classical mechanics and quantum mechanics. The real difference is that in quantum mechanics you have non-commuting observables. If you measure σ_x repeatedly for an electron and take care to keep it isolated from the external world, you always get the same result. But if you then measure σ_z, and get a probabilistic result, when you measure σ_x again, you will again get a probabilistic result the first time. The measurement of σ_z has interfered with the measurement of σ_x. That's because you have non-commuting observables, and those are characteristic of quantum mechanics."

And Dr. Diehard says, "Absolute nonsense! We're big clumsy guys, and when we think we're doing a nice clean measurement of σ_x, we might be messing up all of those hidden observables. Then when we try to measure σ_z, we get a different result, because we've messed things up."[6] And Dr. Diehard continues to say, "My friends the social psychologists tell me that when you do an opinion survey, unless you construct it very carefully, the answers you will get to the questions will depend the order in which they are asked."[7] And he doesn't see any difference between that and measurements of σ_x and σ_z.

Now, thus Dr. Diehard's position. And as John Bell pointed out in the first written of those two articles I cited (which is not the one with the famous inequality): this is, in fact, an irrefutable position, despite all the stuff to the contrary in the literature. On this level, there is no way of refuting it. And he gave a specific example of a classical theory, that on this level reproduced all the results of quantum mechanics: the de Broglie-Bohm pilot wave theory. However, if Dr. Diehard admits one more thing, we can trap him. And I will now explain what that one thing is.

[In Figure 1,] we have a drawing of space and time.[8] Let's consider two measurements on possibly two different systems, done in two regions, $\mathcal{A}$ and $\mathcal{B}$ (forget $\mathcal{B}'$ at the moment, its role will emerge later). These black dots represent substantial regions in spacetime during which an experiment has been conducted. One thing Dr. Diehard will have to admit is that although the results of an experiment in $\mathcal{A}$ may interfere with an experiment in $\mathcal{B}$, the results of an experiment in $\mathcal{B}$ can hardly interfere with the results of an

[6] My friends the anthropologists talk about a lot when they discuss how an anthropologist can affect an isolated society that he or she believes he or she is observing. For some reason I don't understand, they call it the uncertainty principle.

[7] This is true, by the way.

[8] It's really four-dimensional, but due to budgetary constraints, I've had to work with a two-dimensional object.

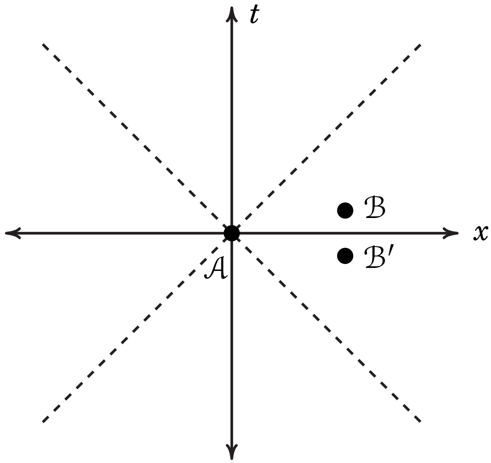

Figure 1: Spacetime. The scale has been chosen so that time t is measured in years, and x in lightyears, and therefore the paths of light rays are 45-degree lines.

experiment in $\mathcal{A}$, unless information can travel backwards in time, which we will assume he does not accept. That's because $\mathcal{A}$ is over and done with and its results recorded in a logbook before $\mathcal{B}$ occurs.

On the other hand, if we imagine another Lorentz observer, with another coordinate system, $\mathcal{B}$ would appear as $\mathcal{B}'$ here. $\mathcal{B}$ and $\mathcal{B}'$, as you can see by eyeball, are on same spacelike hyperbola – there is a Lorentz transformation that leaves $\mathcal{A}$ at the origin of coordinates unchanged and turns $\mathcal{B}$ into $\mathcal{B}'$. $\mathcal{B}$ is spacelike separated from $\mathcal{A}$. A light signal cannot get from $\mathcal{A}$ to $\mathcal{B}$. And nothing traveling slower than the speed of light can get from $\mathcal{A}$ to $\mathcal{B}$. Now that second Lorentz observer would give the same argument I gave, except he would interchange the roles of $\mathcal{A}$ and $\mathcal{B}'$. He would say that the act of doing an experiment at $\mathcal{A}$ cannot interfere with the results of an experiment at $\mathcal{B}'$, because $\mathcal{B}'$ is earlier than $\mathcal{A}$. But $\mathcal{B}'$ is $\mathcal{B}$ – just $\mathcal{B}$ seen by a different observer. Therefore, if you believe in the principle of Lorentz invariance, and if you believe you cannot send information backwards in time, you have to conclude that experiments done spacelike separated locations (sufficiently far apart from each other) cannot interfere with each other. It can't matter what order you ask the questions – if [a] question is being asked of an Earthman and [another] one of an inhabitant of the Andromeda nebula, and they're both being asked today. *We now have a contradiction with the predictions of quantum mechanics for simple systems.*

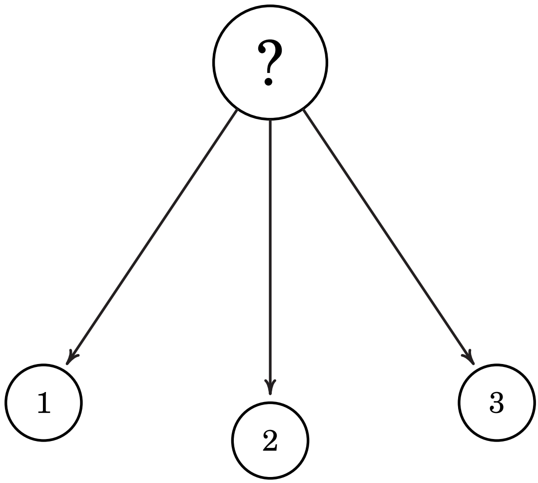

Figure 2: The Diehard Experiment. Regions 1, 2, and 3 are spacelike separated.

Here is the experimental proposal. Figure 2 is a drawing from an imaginary proposal to the Department of Energy for the Diehard Experiment. Three of Dr. Diehard's graduate students are each assigned to an experimental station. As you see from the scale, they are several light minutes from each other. The graduate students, with a lack of imagination, are called numbers 1, 2, and 3.[9] They're informed that once a minute, something will be sent from a mysterious central station to each of the three Diehard teams. What that something is, they don't know. However, they're armed with measuring devices, again whose structure they do not know. They're called Dual Cryptometers, [(see Fig. 3),] because they can measure each of two things, but what those two things are, nobody knows. At least The Diehards don't know. They can turn a switch to either measure A or measure B. They do this decision once a minute, shortly before the announced time of the signal, and sure enough a light bulb lights up that says either A is +1 or A is -1, if they're measuring A. Same thing for B.

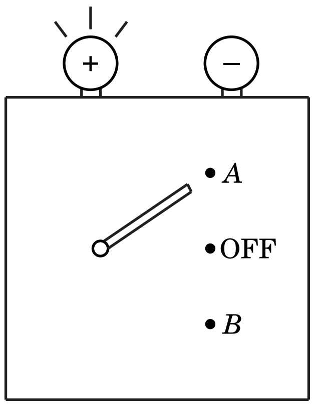

Figure 3: The Acme "Little Wonder" Dual Cryptometer.

[9]They are almost as old as Dr. Diehard. It's difficult to get a thesis under him.

They have no idea what A or B is. It's possible the central station is sending them elementary particles. It's possible the central station is sending them blood samples, which they have the choice of analyzing for either high blood cholesterol or high blood glucose. It is possible the whole thing is a hoax, there is no central station, and a small digital computer inside the cryptometer is making the lights go on and off. They do not know.

In this way, however, they obtain a sequence of measurements, which they record as this:

$$\begin{array}{lll} A_1 = 1 & B_2 = -1 & B_3 = -1 \\ A_1 = 1 & A_2 = -1 & B_3 = 1 \\ B_1 = 1 & B_2 = 1 & A_3 = 1 \\ & \vdots & \end{array}$$

This means observer 1 has decided to measure A and obtained the result +1, observer 2 has decided to measure B and obtained the result -1, observer 3 has decided to measure B and obtained the result -1. And they obtain, in this way, zillions of measurements on a long tape.

They record them in this way because they really *believe* whatever this thing is doing, A_1 *is* +1. That is to say, the value of quantity A that would be measured at station 1 is +1, independent of what is going on at stations 2 and 3, because these three measurements are spacelike separated. That's what they have to believe if they're Diehards. They have to believe there's really some predictable value of this thing which they would know if they knew all the hidden variables, and in this particular case, they don't know what B_1 is, but they know what A_1 is.

As they go through their measurements, they find in that roughly [3/8] of the measurements (they're making random decisions) in which they measure one A and two B's, the [product] is +1: they find that whenever they measure $A_1B_2B_3$, it is +1, and likewise for $B_1A_2B_3$ and $B_1B_2A_3$. Now since they're making their choices at random and since they believe that these things have well-defined meanings, independent of their measurements, they have to believe, if they believe in normal empirical principles, that all the time the [product] of one A and two B's is +1. Sometimes all three of these numbers are +1, sometimes one of them is +1, and two are -1, but the product is always +1. It's as if I gave you a zillion boxes and you opened [3/8] of them, and discovered each of them had a penny in it. You assume within $1/\sqrt{N}$, negligible error, that if you open up all the other boxes, that they would also have pennies in them.

Now by the miracle of modern arithmetic, that is to say, by multiplying $A_1B_2B_3$, $B_1A_2B_3$, and $B_1B_2A_3$ together, and using the fact that each B^2 is +1, *they deduce that if they look on their tape for those experiments in which they've chosen to measure* $A_1A_2A_3$, *they would obtain the answer +1.*

Let's look behind the scenes and see what's actually going on. Well, it's not blood samples we're sending to them, after all. It's three spin-1/2 particles, arranged in the following peculiar initial state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow_1\uparrow_2\uparrow_3\rangle - |\downarrow_1\downarrow_2\downarrow_3\rangle\big). \qquad (7)$$

What A and B are, are simply:

$$A_1 = \sigma_x^{(1)}, \qquad B_1 = \sigma_y^{(1)}, \qquad \text{etc.} \qquad (8)$$

Let's first check that A_1, B_2, and B_3 acting on this state $|\psi\rangle$ is +1:

$$A_1B_2B_3\,|\psi\rangle = \sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}\,|\psi\rangle = |\psi\rangle\,. \qquad (9)$$

Therefore, by the third statement about quantum mechanics that I put on the board in my preliminary section, this quantity is definitely going to be measured to be +1. σ_x turns up into down, σ_y also turns up into down, but with a factor i, or maybe a $-i$, I can never remember, but that's no problem here, because you got two of them, and $(\pm i)^2 = -1$. So, therefore acting on the first component of this state, this operator reproduces the second component, including the minus sign, and acting on the second component it reproduces the first. So this state is indeed an eigenstate with eigenvalue +1. And of course, since everything is permutation invariant, the same thing is true for $B_1A_2B_3$ and $B_1B_2A_3$.

But...

$$A_1A_2A_3 \,|\psi\rangle = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)} \,|\psi\rangle = -\,|\psi\rangle \,. \tag{10}$$

each of the three σ_x's turn an up into a down, without a minus sign. Therefore, this state $|\psi\rangle$ is an eigenstate of $A_1A_2A_3$, but with eigenvalue -1. The Diehards, using only these proto-classical ideas,[10] deduce they will always get $A_1A_2A_3$ to be +1. Sometimes A_1 and A_2 will be -1 and A_3 will be +1, but the product will always be +1. *In fact, if quantum mechanics is right, they will always get -1.*

This is pedagogically superior to the original Bell argument for two reasons. Firstly, it doesn't involve correlation coefficients. It not that classical mechanics says this will happen 47% of the time and quantum mechanics says it will happen 33%. Secondly, it easy to remember. Whenever I lecture on the Bell inequality, I have to look it up again, because I can never remember the derivation. The ingredients in it are so simple that if someone awakens you in the middle of the night four years from now and puts a gun to your

[10]They aren't even so well-developed to as to be called classical physics – they're sort of the underpinnings of classical reasoning.

head and says, "*Show me the GHZM argument!*" you should be able to do it!

What we have shown is that there are quantum mechanical experiments where the conclusions cannot be explained by classical mechanics, even in the most general sense of classical mechanics, unless of course the classical mechanics person is willing to assume transmission of information faster than the speed of light, which, with the relativity principle, is tantamount to transmission of information backwards in time. This, of course, is also John Bell's conclusion, and is, I must say, much misrepresented in the popular literature. And even some not so popular literature. That's not coming out right. I mean some technical literature, where people talk about quantum mechanics necessarily implying connections between spacelike-separated regions of space and time. *That's getting it absolutely backwards.* There are no connections between spacelike separated regions in space and time in this experiment. In fact, there is no interaction Hamiltonian, let alone one that transmits information faster than the speed of the light (except maybe an interaction Hamiltonian between the individual cryptometers and the particles). *It's either quantum mechanics or superluminal transmission of information. Not both!*

Why on earth do people get so messed up, so confused, so wrong about such a simple point?[11] Why do they write long books about quantum mechanics and nonlocality, full of funny arrows pointing in different directions?[12] Because I think they, secretly, in their heart of hearts, they believe

[11]Here now I'm trying to see inside other people's heads, which always a dangerous operation, but let me do it.

[12]That's the technical philosophers, they really...well...I'll avoid the laws of libel.

it's really classical mechanics – we're really putting something over on them – deep, deep down it's really classical mechanics.

[Question from audience member:] *"Isn't part of this whole experiment premised on the assumption that all of these things are outside the light cones of each other or measurements are outside the light cones of each other? So, do you feel that if that premise were to no longer hold, then you couldn't explain what was happening?"* No! I've explained what's happening. Any student who's taken a freshmen course on quantum mechanics and knows what spin is can explain what's happening. I'm saying if they're outside the light cones of each other, there is no *conceivable* classical-mechanical explanation. If they're just far from each other, but not outside of the light cones of each other, you might say there is no *plausible* classical-mechanical explanation. But here this way, there is no *logically possible* classical explanation.

People get things backwards, and they shouldn't. It has been said, and wisely said, that every successful physical theory swallows its predecessors alive. For example, the way statistical mechanics swallowed thermodynamics. In the appropriate domain of experience, the fundamental concepts of thermodynamics – entropy, for example, or heat – were explained in terms of molecular motion. And then we showed that if you define heat in terms of molecular motion, that acted, under appropriate conditions, pretty much the way heat acted in thermodynamics. It's not the other way around. The thing you want to do is *not* interpret the new theory in terms of the old theory, but the old theory in terms of the new.

The other day I was looking at a British video tape of Feynman explaining elementary concepts in science to an interrogator, who I think was the producer of the series, Christopher Sykes, although he wasn't identified and was off screen. He asked Feynman to explain the force between magnets. Feynman hemmed and hawed for a while actually, and then he got on the right track, and said something that was dead on the nail. He said, you got it all backwards, because you're not asking me to explain the force between the seat of your pants and the seat of a chair. You want me, when you say, "Explain the force between magnets," to explain the force in terms of the kinds of forces that *you think* of as being fundamental: those between bodies in contact.[13] Anyways, whereupon we physicists all know, it's the other way around: the fundamental force between atoms is the electromagnetic force (which does fall off as $1/r^2$). Where Christopher Sykes was confused was he was asking something impossible: to explain the force between magnets in terms of the pants-chair force, to explain the fundamental quantity in terms of the derived one.

Likewise, a similar error is being made here. *The problem is not the interpretation of quantum mechanics. That is getting things just backwards. The problem is the interpretation of classical mechanics.*

[13] Obviously, I'm not phrasing it as wonderfully as Feynman would. Well, as Picasso said in another circumstance, "It doesn't have to be a masterpiece for you to get the idea."

The Return of Schrödinger's Cat

I'm going to address this in particular with the famous (or infamous) projection postulate. The fundamental analysis is von Neumann:

> J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*. (1932).

I don't read two words of German, but I wanted to put the down the early publication. I read an English translation. The position I'm going to advocate is associated with Hugh Everett, in a classic paper:

> H. Everett III, *Rev. Mod. Phys.* **29** (1957) 434 ,

and some of the things I'll say about probability later come from a paper Jim Hartle:

> J. Hartle, *Am. J. Phys.* **36** (1968) 704,

and one by Cambridge's own Eddie Farhi, Jeffrey Goldstone, and Sam Gutmann:

> E. Farhi, J. Goldstone, S. Gutmann, *Ann. Phys.* **192** (1989) 368.

von Neumann's Model of Measurement

I'd like to begin by recapitulating von Neumann's analysis of the measurement chain. I prepare an electron in a σ_x eigenstate:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + |\downarrow\rangle\Big), \tag{11}$$

and I measure σ_z. The famous nondeterministic "reduction of the wave function" takes place, and with equal probabilities, I cannot tell which, the spin either goes up or down:

$$|\psi\rangle \longrightarrow \begin{cases} |\uparrow\rangle \\ |\downarrow\rangle \end{cases} \quad \text{equal probabilities.} \tag{12}$$

But this is rather unrealistic, even for a highly-idealized measurement. An electron is a little tiny thing and I have bad eyes. I probably won't be able to see directly what its spin is. There has to be an intervening measuring device. So, we complicate the system. The initial state is the same as before, as far as the electron, but the measuring device is in some neutral state:

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow, M_0\rangle + |\downarrow, M_0\rangle\Big). \tag{13}$$

The electron interacts with the measuring device, von Neumann showed us how to set things up, so that with the interaction Hamiltonian, if the electron is spinning up (the measuring device is maybe one of those dual cryptometers), the light bulb saying "plus" flashes, and if the electron is spinning down, the light bulb saying "down" flashes:

$$|\psi_f\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow, M_+\rangle + |\downarrow, M_-\rangle\Big). \tag{14}$$

This is normal deterministic time evolution according to Schrödinger's equation. Now I come by, I can't see the electron, but I observe the device. By the usual projection postulate, I either see it in state plus or state minus. I make the observation. If I see it in state plus, the rest of the wave function is annihilated, crossed out, and I get with equal probabilities:

$$|\psi\rangle \longrightarrow \begin{cases} |\uparrow, M_+\rangle \\ |\downarrow, M_-\rangle \end{cases} \quad \text{equal probabilities.} \tag{15}$$

The result is the same as before, because the electron is entangled with the device. I measured the device, the electron comes along for the ride.

Now let's complicate things a bit. Let's suppose, however, I cannot do the measurement because I'm giving this lecture. However, I have a colleague, a very clever experimentalist (for purposes of definiteness, lets say it's Paul Horowitz), who has constructed an ingenious robot. I'll call him Gort.[14] He constructed this robot, and I say, "Gort, I want you during the lecture to go and see what the measuring device says about the electron." So Gort comes, and does this, and of course, although he's an extremely ingenious and complicated robot, he's just a big quantum mechanical system like anything else. This is the same story: things starts out with the electron up, measuring device neutral, a certain register in a RAM chip inside Gort's belly also has nothing written on it. Then everything interacts, and the state of this world is electron up, measuring device says up, Gort's RAM chip's register says up, plus the same thing with up replaced by down, all divided by $\sqrt{2}$. And Gort comes rolling in the door there on his rollers. And I say, "Hey Gort, which way is the electron's spinning?" And he tells me it,

[14] It's a good name for a robot.

and *Whammo!* It either goes into one or the other of these states with 50% probability. But Gort is very polite. He observes that I am lecturing, so rather than coming to me directly, he rolls up to my colleague, Prof. Nelson, sitting there in the corner, and hands him a slip of print out, and it says either "up" or "down," and "Pass this on to Sidney when the lecture is over." And he rolls away.

Well, of course, vitalism was an intellectually alive position in the early 19th century. Dr. Lydgate in *Middle-March*[15] [held] that living creatures are not simply complicated mechanical systems, but it hasn't had many advocates this century, and I think most of us would admit that David is just another quantum mechanical system, although, perhaps more complicated than the electron and Gort.[16] It's the same story as before, the state of the world after all this has happened is: electron up, measuring device says up, Gort's RAM chip says up, David's slip of paper says [up, and David's] mind has thought up, plus the same thing with down, divided by $\sqrt{2}$. After the lecture I go up to him and say: "What's up David?" *Whammo!* He tells me and the whole wave function collapses.

Now this is getting a little silly. Especially if you consider the possibility that, after all, I'm getting on in years, I'm not in perfect health, here I am running around a lot, maybe I have a heart attack before the lecture is over and die. What happens then? Who reduces the wave packet? Yakir Aharonov, who has of course since acquired great fame for himself, was a young postdoc at Brandeis when I was a young postdoc at Harvard, and I had been reading von Neumann and thinking about this and had come to a conclusion which I did not like, which was solipsism. I was the only

[15] Which will be appearing on TV tomorrow.

[16] And certainly more likable, but anyway.

creature in the world that could reduce wave packets, otherwise it didn't make sense. I was not totally happy with this position, even though I was as egotistical as any young man, and indeed probably more egotistical than most, I was still unhappy with the position. I was discussing this with Aharonov. Even in his youth, he would smoke these enormous cigars, which he would use to punctuate the conversation – he would take huge drafts on them.[17] Anyway, I explained this position, and he said, "I see," [and after taking a huge draft on his cigar, he asked,] "Tell me, before you were born, could your father reduce wave packets?"

I will argue that, in fact, there is:

NO special measurement process
NO reduction of the wave function
NO indeterminacy
NOTHING probabilistic

in quantum mechanics – only deterministic evolution according to Schrödinger's equation. This is not a novel position. In the famous paper on the cat, Schrödinger raises this position and instantly that the cat is, in fact, a coherent superposition of being dead and being alive, and said, "We reject the ridiculous possibility." Some years later in the paper on Wigner's friend, where Wigner attempted to resolve the ancient mind-body problem through the quantum theory of measurement, he also raised this position, and said it was "absurd." There is a recent paper by Zurek in Physics Today (Zurek has made major contributions to the theory of decoherence) where instead of just saying it's ridiculous or absurd, he actually raised a question one can talk about. He said: if this is so, why do I, the observer, perceive only one of the outcomes? This is now the question I will attempt

[17] He was, and is, sort of the quantum George Burns.

to address: Zurek's question. If there is no reduction of the wave packet, why do I feel, at the end of the day, that I have observed a definite outcome that the electron is spinning up or the electron is spinning down?

Mott's Cloud Chambers

In order to ease into this, I'd like to begin with an analysis of Nevill Mott's. Nevill Mott worried, way back when in 1930, about cloud chambers. He said: look, an atom releases an ionizing particle in the center of a cloud chamber in an S-wave. And it makes a straight-line track. Why should it makes a straight-line track? If I think about an S-wave, it's spherically symmetric, why didn't I get some spherically-symmetric random distribution of sprinkles? Why should the track be a straight line? We're going to answer that question in a faster and slicker way than Nevill Mott did, although we have the advantage, of course, of 64 years of hindsight.

We must assume that the scattering between the particle and an atom, when it ionizes it, is unchanged, or changed only within some small angle. Otherwise, even classically, the particle would bounce around like a pinball on a pinball table. Let $|C\rangle$ be the state of the cloud chamber. We define a "linearity operator" L, a projection operator, so that:

$$L\,|C\rangle = |C\rangle\,, \tag{16}$$

if there is a track and if it forms a straight line, to within some small angle, and

$$L\,|C\rangle = 0, \tag{17}$$

if the track is not a straight line, or if there is no track for that matter. Let's imagine we start out the problem in some initial state, where the particle is concentrated near the center of the position and near some momentum **k**, and the cloud chamber is in a neutral state $|C_0\rangle$, unionized, ready to make tracks. This evolves into some final state:

$$|\psi_i\rangle = |\Phi_{\mathbf{k}}, C_0\rangle \longrightarrow |\psi_{f\mathbf{k}}\rangle \,. \tag{18}$$

We all believe that if you started the particle in a narrow beam, it would of course make a straight line track along that beam. There would be a track, it would be an eigenstate of L, and it would have eigenvalue +1:

$$L\,|\psi_{f\mathbf{k}}\rangle = |\psi_{f\mathbf{k}}\rangle \,. \tag{19}$$

Here comes the tricky part (not tricky to follow, but tricky clever). Consider an initial state that's an integral over the angles of **k**:

$$|\psi_i\rangle = \int d\Omega_{\mathbf{k}}\,|\Phi_{\mathbf{k}}, C_0\rangle \,. \tag{20}$$

This is a state where the particle is initially in an S-wave, and the cloud chamber is still in the neutral state – that's independent of **k**. That evolves by the causal linearity of Schrödinger's equation into the corresponding superposition of these final states:

$$|\psi_i\rangle \longrightarrow |\psi_f\rangle = \int d\Omega_{\mathbf{k}}\,|\Phi_{f\mathbf{k}}\rangle \,. \tag{21}$$

But if I have a linear superposition of the eigenstate of the operator L, each of which is an eigenstate with eigenvalue +1, then the combination is also an eigenstate with eigenvalue +1:

$$L\,|\psi_f\rangle = |\psi_f\rangle \,. \tag{22}$$

So, this also has straight line tracks in it.

That's the short version of Mott's argument. Mott said the problem is that people think of the Schrödinger equation as a wave in three-dimensional space, rather than a wave in a multidimensional space. I would phrase that, making a gloss on this,[18] and say: *the problem is that people think of the particles as a quantum-mechanical system, but the cloud chamber as a classical-mechanical system. If you're willing to realize that both the particle and the cloud are two interacting parts of one quantum-mechanical system, then there's no problem.* It's an S-wave, not because the tracks are not straight lines, but because there is a rotationally-invariant correlation between the momentum of the particle and where the straight line points. But it's always an eigenstate of L. Nobody doubts it, the tracks in cloud chambers, or bubble chambers (if you're young enough), are straight lines, even if the initial state is an S-wave.

Albert's Argument

I will now give an argument due to David Albert, and I'll return to Zurek's question. Zurek asked, "Why do I always have the perception that I have observed a definite outcome?" To answer this question, no cheating, we can't assume Zurek is some vitalistic spirit loaded with *élan vital*, unobeying the laws of quantum mechanics. We have to say the observer (I don't want to make it Zurek, that's using him without his permission – I'll make it me, Sidney) has some Hilbert space of states, $|S\rangle \in \mathscr{H}_S$, and some condition in Sidney's consciousness corresponds to the perception that

[18] He's dead, so I can't check whether this it's an accurate phrasing.

he observed a definite outcome. So there's some projection operator on it: the definiteness operator, D. If you want, we can give it an operational definition. For the state where the definiteness operator is +1:

$$D\,|S\rangle = |S\rangle\,, \tag{23}$$

is one where a (hypothetical) polite interrogator asks Sidney, "Have you observed a definite outcome?" and he says, "Yes." In the orthogonal states:

$$D\,|S\rangle = 0, \tag{24}$$

he would say, "No, gee, I was looking someplace else when that sign flashed on," or "I forgot," or "Don't bother me man, I'm stoned out of my mind!" or any of those things.

Let's begin with our same old system as before: electron-measuring apparatus, and Sidney:

$$|\psi_i\rangle = |\uparrow, M_0, S_0\rangle\,. \tag{25}$$

If the electron is spinning up, the measuring apparatus measures spin in the up direction, we get a definite state, no problem of superposition:

$$|\psi_i\rangle \longrightarrow |\psi_f\rangle = |\uparrow, M_+, S_+\rangle\,, \tag{26}$$

and Sidney thinks, "I've observed a definite outcome."

$$D\,|\psi_f\rangle = |\psi_f\rangle\,. \tag{27}$$

The measurement is up. Also if everything is down:

$$|\psi_i\rangle = |\downarrow, M_0, S_0\rangle \longrightarrow |\psi_f\rangle = |\downarrow, M_-, S_-\rangle\,, \tag{28}$$

and

$$D\,|\psi_f\rangle = |\psi_f\rangle\,. \tag{29}$$

What if we start out with a superposition? Same story as Nevill Mott's cloud chamber:

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow, M_0, S_0\rangle + |\downarrow, M_0, S_0\rangle\Big), \tag{30}$$

$$|\psi_i\rangle \longrightarrow |\psi_f\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow, M_+, S_+\rangle + |\downarrow, M_-, S_-\rangle\Big), \tag{31}$$

Same reason the cloud chamber always observes a track to be a straight line, Sidney always has the feeling he's observed a definite outcome:

$$D\,|\psi_f\rangle = |\psi_f\rangle\,. \tag{32}$$

[Comment from audience member, jokingly:] *"Which one of them was up or down?"* No, that's not what Zurek said. Zurek didn't say it's a matter of common experience that in this experiment we always observe the electron spinning up. And Nevill Mott didn't say it was a matter of common experience that in the cloud chamber the straight line is always pointing along the z-axis. The matter of common experience is that Sidney always has the perception that he has observed a definite outcome, if you set up the initial conditions correctly. The matter of common experience is that the cloud chamber is always in a straight line.

If you don't like [Mott's argument], you can't like [Albert's]. If you like [Mott's argument], you have to like [Albert's]. The problem here, the confusion Nevill Mott removed, was refusing to think of the cloud chamber as a quantum mechanical system. The problem here is refusing to think of Sidney as a quantum mechanical system.

Probability

I will go on to discuss the question of probability. Probability is a difficult question to discuss, because it requires, from the word go, that we look at something counterfactual. If I ask whether a given sequence is or is not random, I can't do that even in classical probability theory for a finite sequence. For example, if I consider a binary sequence, where the entries are either 1 or -1, I say, is the sequence 1 a random sequence? Obviously, there's no way of answering that question. For a sequence of an Avogadro number of digits, it's logically no easier. But, if I have an infinite sequence, I *can* ask whether it's random, and let me talk about that.

Let me suppose I have an infinite sequence of +1's and -1's, which I might think represent heads and tails, and I want to see if these sequences can be interpreted as a fair coin flip. Well, firstly I want the average value of the quantity σ_r, $r = 1, 2, \cdots$, which, of course, is simply the limit of the average value of the first N terms as $N \to \infty$, to converge to zero. We have a sequence of independent random flips of a fair coin if:

$$\overline{\sigma} = \lim_{N\to\infty} \overline{\sigma}^N = \lim_{N\to\infty} \frac{1}{N} \sum_{r=1}^{N} \sigma_r = 0. \tag{33}$$

Also if I were an experimenter, I would probably look at correlations. I would take the value of σ_r, times the value of σ_{r+a}, for some value of a, look at the limit of this correlation, and ask that this quantity be also zero, for any value of a:

$$\overline{\sigma}^a = \lim_{N\to\infty} \overline{\sigma}^{N,a} = \lim_{N\to\infty} \frac{1}{N} \sum_{r=1}^{N} \sigma_r \sigma_{r+a} = 0. \tag{34}$$

That way I would reject sequences like 1, 1, -1, -1, 1, 1, -1, -1, $\cdots$, which nobody would call random. And I also look for triple

$$\lim_{N\to\infty} \frac{1}{N} \sum_{r=1}^{N} \sigma_r \sigma_{r+a} \sigma_{r+b} = 0, \tag{35}$$

and higher correlations. And if all those things were zero, then I'd say it's a pretty good chance it's a random sequence. Actually, I would be sloppy in the way experimenters are sloppy. I would actually have to provide even further tests if I wanted the real definition of randomness (the Martin-Löf definition of randomness), but this will be good enough.

We want to ask the parallel question in quantum mechanics. We start out with an electron in a state I'll call "sidewise":

$$|{\rightarrow}\rangle = \frac{1}{\sqrt{2}}\Big(|{\uparrow}\rangle + |{\downarrow}\rangle\Big), \tag{36}$$

which is just our good old σ_x eigenstate, the same state I've used before. I consider an infinite sequence of electrons heading toward my σ_z measuring apparatus:

$$|\psi\rangle = |{\rightarrow}\rangle \otimes |{\rightarrow}\rangle \otimes |{\rightarrow}\rangle \cdots \tag{37}$$

And I do the usual routine with the measuring system and Sidney's head and turn it into a sequence of memories in Sidney's head, or maybe Sidney has a notebook and he writes +1, -1, +1, -1, $\cdots$. I obtain a sequence of records, correlated with the z component of spin. And I ask, "Does this observer perceive a sequence of independent random flips?" Well, we know it's all correlated with σ_z, so in order to keep the transparency from overflowing its boundaries, I just looked at σ_z, rather than operators for the records. I define the average value of σ_z as exactly the same way as before, and then I say, is $|\psi\rangle$ an eigenstate of

$$\overline{\sigma}_z = \lim_{N\to\infty} \overline{\sigma}_z^N = \lim_{N\to\infty} \frac{1}{N} \sum_{r=1}^{N} \sigma_z^{(r)}, \tag{38}$$

with eigenvalue zero?

If it is, despite the fact that there's nothing probabilistic in here, we can say that the average value of σ_z is guaranteed to be observed to be zero. The calculation is sort of trivial. Let's compute the norm of the state obtained by applying this $\overline{\sigma}_z$ to $|\psi\rangle$:

$$\overline{\sigma}_z \, |\psi\rangle \, . \tag{39}$$

Well, it's two sums:

$$\left| \overline{\sigma}_z \, |\psi\rangle \, \right|^2 = \frac{1}{N^2} \, \langle\psi| \sum_{r=1}^{N} \sum_{s=1}^{N} \sigma_z^{(r)} \sigma_z^{(s)} \, |\psi\rangle \, . \tag{40}$$

In this particular state, of course, if r is not equal to s, this "expectation value" is equal to zero, because you just get the product of the independent expectation values, which are individually zero. On the other hand, if r is equal to s, then this is σ_z^2 squared, which we all know is +1:

$$\langle\psi| \, \sigma_z^{(r)} \sigma_z^{(s)} \, |\psi\rangle = \delta^{rs} \, . \tag{41}$$

Therefore, the double sum collapses to a single sum, only the terms with $r = s$ contribute, and each entry with $r = s$ contributes 1, so you get N. Thus the result is:

$$\lim_{N\to\infty} \left| \overline{\sigma}_z \, |\psi\rangle \, \right|^2 = \lim_{N\to\infty} \frac{1}{N^2} \, N = 0. \tag{42}$$

And, of course, the same thing happens for all those correlators, because each one is the sum of terms with a $1/N$ in front, and only the entries that match perfectly will give you a nonzero contribution.

So, this definite quantum mechanical-state, completely determined by the initial conditions, nevertheless matches this experimenter's (also considered a quantum mechanical system) definition of randomness. *Something that would be impossible in classical mechanics, but it's quantum mechanics, stupid!*

Now, one final remark. In Tom Stoppard's play *Jumpers*, there's an anecdote about the philosopher Ludwig Wittgenstein. I have no idea whether it's a real story or a Cambridge folk story. Anyways, it goes like this: a friend is walking down the street in Cambridge and sees Wittgenstein standing on the street corner, lost in thought. He said, "What's bothering you, Ludwig?" And Wittgenstein says, "I was just wondering why people said it was *natural* to believe the sun went around the earth, rather than the other way around." The friend says, "Well, that's because it looks like the sun goes around the earth." And Wittgenstein thinks for a moment, and he says, "Tell me, what would it have looked like if it had looked like it was the other way around?"

People say the reduction of the wave packet occurs because it looks like the reduction of the wave packet occurs. And that is indeed true. *What I am asking you, in the second main part of this lecture, is to consider seriously what it would look like if it were the other way around: if all that ever happened were causal evolution according to quantum mechanics. And what I have tried to convince you is that what it looks like is ordinary everyday life.*

Welcome home.

Thank you for your patience.