The Gas Guzzler

Moral: In Newtonian dynamics, you can have arbitrarily large masses, but they cannot be infinitely large. Otherwise, you run into problems with Galileo's principle of relativity.

Galileo's principle of relativity states that the laws of physics are the same in all inertial reference frames. Put more prosaically, the truly real physical quantities are those that every passive observer would agree on, so long as those observers are moving at constant speeds. Let this sink in. This principle of relativity makes a radial point: you can't measure an object's "true" speed. There is no such thing as a "true" speed. You only measure an object's speed *relative to something else.* This might seem trivial, but it is hard to overstate how huge an idea this was, as it flies in the face of our everyday experience. (In fact, I suspect this is a major contributing factor as to why there were almost 2,000 years between Aristotle and Galileo.) The following problem confronts this topic head-on, in hopes of making you more sympathetic to why the principle of relativity is anything but obvious to the ancient Greeks.

A truck with mass m is driving down a straight road at an initial velocity $v_i = 60$ mph. For simplicity, assume there is no friction, so it requires no work for the truck to maintain a constant speed. To accelerate, it must guzzle ℓ gallons of gas in order to increase its kinetic energy by 1 Joule.

(a) How many gallons of gas must the truck guzzle to increase its speed from $v_i = 60$ mph to $v_f = 80$ mph? Feel free to ignore the change in the mass of the truck due to burning the fuel.

(b) Now imagine you're in a car, also going 60 mph, driving alongside the truck. In your reference frame, the speed of the truck is $v_i' = 0$ before it accelerates, and $v_f' = 20$ mph after it accelerates. The primes here are to indicate that they're the speeds in a different reference frame than in part (a). If you repeat the calculation in part (a), but in this new reference frame, how many gallons would you calculate that the truck guzzled?

(c) You should have gotten a different answer for parts (a) and (b). On face value, this doesn't make sense. Surely the amount of gas guzzled by the truck shouldn't depend your speed – you're just an observer. What are we missing in order to resolve this paradox? If you want to think about it, stay on this page. The final part of this problem (continued on the next page) contains a spoiler.

(d) The resolution is that we're neglecting the rotation of the earth! Show that if you don't neglect the fact that the truck pushes back on the earth while accelerating, then the amount of gas it guzzles is the same in all inertial reference frames. To make the math easier without losing the moral of the story, feel free to pretend the earth is a giant block of mass M, and this system is one dimensional.

This is a version of a problem from Walter Appel's textbook.